Key

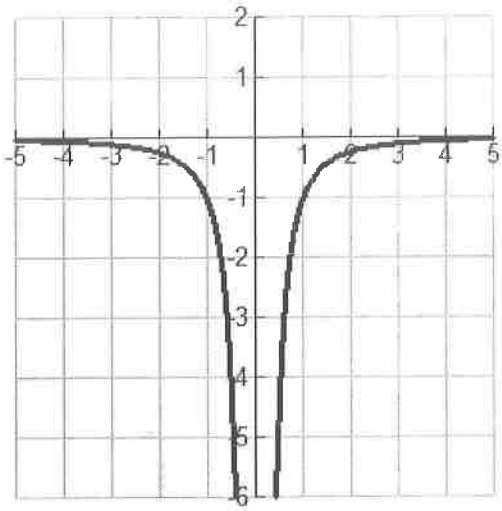
Math 4

Name _____

Semester 1 Exam Review

1. **Function Families**

Given a function or graph, find the domain, range, symmetries, degree and type of function family.



Domain: $\{x : x \neq 0\}$

Range: $\{y : y < 0\}$

Symmetry: Even → Reflection over y-axis

Type of function: Inverse variation $y = \frac{k}{x^2}, k < 0$

2. **Function Operations**

Given two functions, calculate arithmetic operations and the composition of the functions.

$t(x) = 3x - 10$ $\quad$ $j(x) = x^2 + 2x + 5$

Calculate the following:

$[t + j](x)$

$= 3x - 10 + x^2 + 2x + 5$

$= \boxed{x^2 + 5x - 5}$

$[t - j](x)$

$= 3x - 10 - (x^2 + 2x + 5)$

$= 3x - 10 - x^2 - 2x - 5$

$= \boxed{-x^2 + x - 15}$

$t(x) \cdot j(x)$

$= (3x - 10)(x^2 + 2x + 5)$

$= 3x^3 + 6x^2 + 15x - 10x^2 - 20x - 50$

$= \boxed{3x^3 - 4x^2 - 5x - 50}$

$j(t(x))$

$= j(3x - 10)$

$= (3x - 10)^2 + 2(3x - 10) + 5$

$= 9x^2 - 60x + 100 + 6x - 20 + 5$

$= \boxed{9x^2 - 54x + 85}$

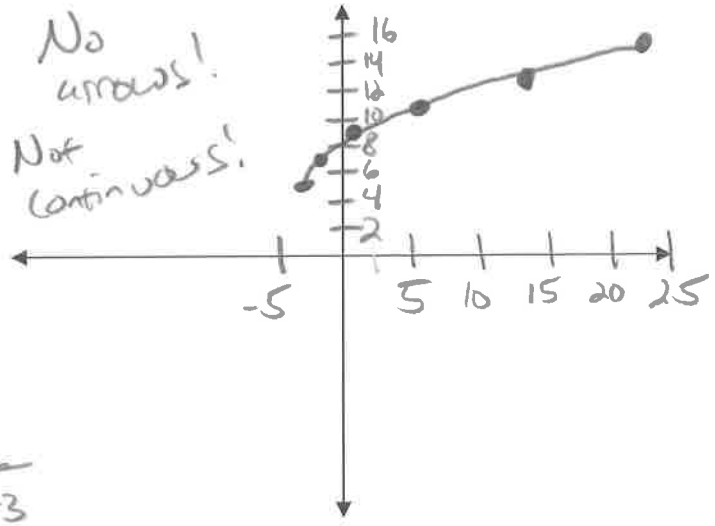
3. **Parametric Equations**

Use parametric equation to construct a graph. Convert parametric equations to rectangular form.

$$\begin{cases} x(t) = t^2 - 3 \\ y(t) = 2t + 5 \end{cases}$$

Graph the equations over the interval $0 \le t \le 5$ then convert the equations to rectangular form & simplify.

t	x	y
0	-3	5
1	-2	7
2	1	9
3	6	11
4	13	13
5	22	15



Convert

$x = t^2 - 3$

$x + 3 = t^2$

$t = \pm\sqrt{x+3}$

$y = 2(\sqrt{x+3}) + 5 = 2\sqrt{x+3} + 5$

$y = 2(-\sqrt{x+3}) + 5$

$y = -2\sqrt{x+3} + 5$

4. **Chunking/*u*-substitution**

Solve for *x*.

$5e^{2x} - 9e^x - 10 = 2e^{2x} + 4e^x$

$u = e^x$

$5u^2 - 9u - 10 = 2u^2 + 4u$

$3u^2 - 13u - 10 = 0$

$(3u+2)(u-5) = 0$

$u = \frac{-2}{3}, u = 5$

$e^x = -\frac{2}{3}$ → No solution

$e^x = 5$

$\ln 5 = x$

$x \approx 1.609$

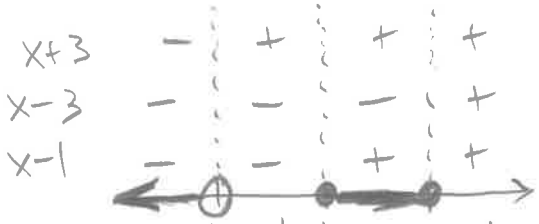
5. **Number Line Analysis**

Solve the inequality below.

$$\frac{x^2 - 4x + 3}{x+3} \le 0$$

$\frac{(x-3)(x-1)}{x+3} \le 0$

Remember: x can never be $= -3$!!



Interval Notation

$x < -3$ or $1 \le x \le 3$	$(-\infty, -3) \cup [1, 3]$

6. **Arithmetic and Geometric Sequences and Series including Sigma Notation**

Write out what is meant by $\sum_{k=1}^{4} \frac{1}{(2k+1)(2k+3)}$.

$= \frac{1}{(2(1)+1)(2(1)+3)} + \frac{1}{(2(2)+1)(2(2)+3)} + \frac{1}{(2(3)+1)(2(3)+3)} + \frac{1}{(2(4)+1)(2(4)+3)}$

Express each of the following in sigma notation:

(a) $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$

(b) $-1+2-3+4-5+6-\ldots+20$

a) $\sum_{k=1}^{5} \frac{1}{k}$ b) $\sum_{k=1}^{20} (-1)^k \cdot k$

Find the 12th term of the sequence 3, 8, 13, 18, ...

$a_{12} = 3+5(12-1)$

$= 3+55 = \boxed{58}$

Fill in the missing blanks of the arithmetic sequence 9, 16, 23, 30, 37

$37 = 9+4d$

$28 = 4d$

$7 = d$

7. **Matrix operations**

Simplify each.

$$\begin{bmatrix} -1 & -1 \\ -6 & 3 \end{bmatrix} + \begin{bmatrix} -5 & -1 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 6 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} -17 & -37 \\ -16 & -9 \end{bmatrix} \quad (2\times2)\ \begin{bmatrix} 1 & -6 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 3 \\ -2 \end{bmatrix} \quad (2\times1)\ (4\times1)$$

= No solution / Not possible

Use calc! Hit button to right of "nine" key.

Must be same dimension to add/subtract!

Use matrices to solve the system.

$$\begin{cases} -4x-15y=-17 \\ -x+5y=-13 \end{cases}$$

$$\begin{bmatrix} -4 & -15 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -17 \\ -13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 & -15 \\ -1 & 5 \end{bmatrix}^{-1} \begin{bmatrix} -17 \\ -13 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}}$$

8. **Rational Functions**

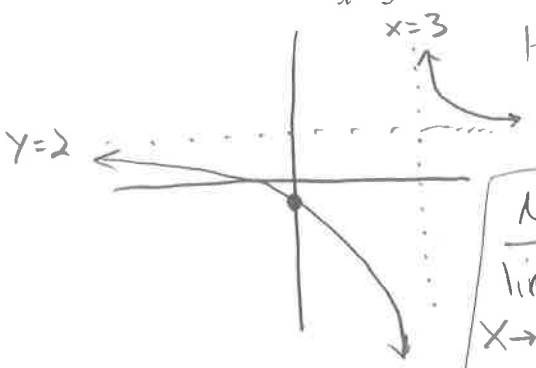
1. Use limit notation to describe the behavior of $j(x)$ near its vertical and horizontal asymptotes. It would also be nice if you could draw a sketch of the function as well.

y-int: $-\frac{5}{3}$

$j(x)=\frac{2x+5}{x-3}$

Vert asym: $x=3$

Horiz. asym: $y=2$



Near vertical asymptote:

$\lim_{x\to 3^-} j(x) = -\infty$

$\lim_{x\to 3^+} j(x) = \infty$

Near horizontal:

$\lim_{x\to\pm\infty} j(x) = 2$

→ Can write this as two separate limits.

2. Add, subtract, multiply or divide the rational expressions below. Simplify your answer.

a. $\frac{x^2-x-6}{2x^2+9x+4}\cdot\frac{x^2-16}{2x^2-7x-4}$

$=\frac{(x-3)(x+2)}{(2x+1)(x+4)}\cdot\frac{(x+4)(x-4)}{(2x+1)(x-4)}$

$=\boxed{\frac{(x-3)(x+2)}{(2x+1)^2}}$

b. $\frac{3x-4}{2x^2+3x+1}+\frac{5}{2x+1}\cdot\frac{x+1}{x+1}$

$(2x+1)(x+1)$

$=\frac{3x-4+5x+5}{(2x+1)(x+1)}$

$=\boxed{\frac{8x+1}{(2x+1)(x+1)}}$

c. $\frac{2(x+3)^2}{x-3}\div\frac{4}{x^2-9}$

$=\frac{\cancel{2}(x+3)(x+3)}{(x-3)}\cdot\frac{(x+3)(x-3)}{\cancel{4}_2}$

$=\boxed{\frac{(x+3)^3}{2}}$

d. $\frac{6x+5}{2x+3}-\frac{2x-1}{2x+3}$

$=\frac{6x+5-(2x-1)}{2x+3}$

$=\frac{4x+6}{2x+3}$

$=\frac{2(2x+3)}{2x+3}=\boxed{2}$

Determine the values of the properties below. Write "none" if one does not exist. The domain is for the original function.

3. $f(x)=\dfrac{(2x-3)(x+5)}{x^2+6x+5}$ $=\dfrac{2x-3}{x+1}$

$(x+5)(x+1)$

Domain: $x\neq -5, x\neq -1$

x-intercept(s): $(3/2, 0)$

y-intercept: $(0, -3)$

horizontal asymptote(s): $y=2$

vertical asymptote(s): $x=-1$

oblique asymptote: None

Deg. num one bigger than deg. denom.

hole: $(-5, \frac{13}{4})$

$f(-5)=\dfrac{2(-5)-3}{-5+1}=\dfrac{-13}{-4}$

4. $w(x)=\dfrac{2x^2-13x-45}{3x^3+28x^2+9x}$ $=\dfrac{(2x+5)(x-9)}{x(3x+1)(x+9)}$

$x(3x^2+28x+9)$

Domain: $x\neq 0, x\neq -\frac{1}{3}, x\neq -9$

x-intercept(s): $(-\frac{5}{2}, 0)$ & $(9, 0)$

y-intercept: None

horizontal asymptote(s): $y=0$

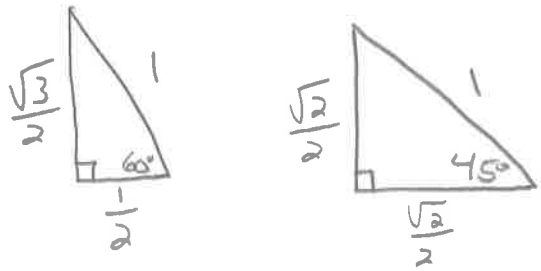
vertical asymptote(s): $x=0, x=-\frac{1}{3}, x=-9$

oblique asymptote: None

hole: None

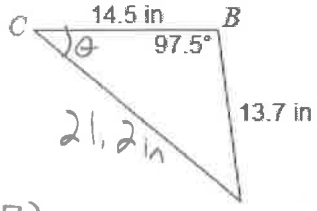
9. **Unit Circle**

Yeah, know all the values around the Unit Circle.



10. **Law of Sines and Cosines**

Find $m\angle C$.



$b^2=14.5^2+13.7^2-2(14.5)(13.7)\cos 97.5$

$b^2\approx 449.798$

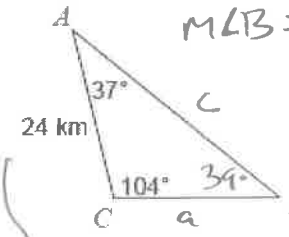
$b\approx 21.2$

$\dfrac{\sin C}{13.7}=\dfrac{\sin 97.5}{21.2}$

$\sin C=\dfrac{13.7\cdot\sin 97.5}{21.2}$ ≈ 0.641

$C\approx 39.8°$

Solve all missing sides and angles.



$m\angle B=39°$

$\dfrac{a}{\sin 37}=\dfrac{24}{\sin 39}$

$a=\dfrac{24\cdot\sin 37}{\sin 39}$

$a\approx 23$ km

$\dfrac{c}{\sin 104}=\dfrac{24}{\sin 39}$

$c=\dfrac{24\cdot\sin 104}{\sin 39}$

$c\approx 37$ km

11. **Trigonometric Simplification**

Remember: $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$, $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$ and $\sin^2 x + \cos^2 x = 1$

*****As always, study your worksheets and tests/quizzes in addition to working through the problems in this review.**